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## Erratum

# Erratum to “Iterative Methods for Variational Inequalities over the Intersection of the Fixed Points Set of a Nonexpansive Semigroup in Banach Spaces”

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In my recent published paper [1] to prove Lemmas 3.1 and 5.1, an inequality involving the single-valued normalized duality mapping  $J$  from  $X$  into  $2^{X^*}$  has been used that generally turns out there is no certainty about its accuracy. In this erratum we fix this problem by imposing additional assumptions in a way that the proofs of the main theorems do not change.

We recall that a uniformly smooth Banach space  $X$  is  $q$ -uniformly smooth for  $q > 1$  if and only if there exists a constant  $\beta_q > 0$  such that, for all  $x, y \in X$ ,

$$\|x + y\|^q \leq \|x\|^q + q\|x\|^{q-2}\langle y, J(x) \rangle + 2\beta_q\|y\|^q, \quad (1)$$

for more details see [2]. Therefore, if  $q = 2$ , then there exists a constant  $\beta > 0$  such that

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, J(x) \rangle + 2\beta\|y\|^2. \quad (2)$$

It is well known that Hilbert spaces,  $l_p$  and  $L_p$  for  $p \geq 2$ , are 2-uniformly smooth.

Throughout the paper we suggest to impose one of the following conditions:

- (a) the Banach space  $X$  is 2-uniformly smooth;
- (b) there exists a constant  $\beta \in \mathbb{R}^+$  for which  $J$  satisfies the following inequality:

$$\langle y, J(x+y) \rangle \leq \langle y, J(x) \rangle + \beta \|y\|^2, \quad (3)$$

for all  $x, y \in X$ .

*Remark 1.1.* If  $J$  is  $\beta$ -Lipschitzian, then  $J$  satisfies (3) and is norm-to-norm uniformly continuous that suffices to guarantee that  $X$  is 2-uniformly smooth. For more results concerning  $\beta$ -Lipschitzian normalized duality mapping see [3].

Note that since every uniformly smooth Banach space  $X$  has a Gateaux differentiable norm and each nonempty, bounded, closed, and convex subset of  $X$  has common fixed point property for nonexpansive mappings, we have  $D(x_n) \cap C \neq \emptyset$  in [1]. So, when  $X$  is 2-uniformly smooth, we can remove these two conditions from Theorems 3.2, 4.2, and 5.2 in [1].

Considering the above discussion to complete our paper, we reprove Lemmas 3.1 and 5.1 of [1] here with some little changes.

**Lemma 3.1** (see [1]). *Either let  $X$  be a real Banach space, and let  $J$  be the single-valued normalized duality mapping from  $X$  into  $2^{X^*}$  satisfying (3) or let  $X$  be a 2-uniformly smooth real Banach space. Assume that  $F : X \rightarrow X$  is  $\eta$ -strongly monotone and  $\kappa$ -Lipschitzian on  $X$ . Then*

$$\psi(x) = I(x) - \mu F(x) \quad (4)$$

*is a contraction on  $X$  for every  $\mu \in (0, \eta/\beta\kappa^2)$ .*

*Proof.* If  $J$  satisfies (3), considering the inequality

$$\|x+y\|^2 \leq \|x\|^2 + 2\langle y, J(x+y) \rangle, \quad (5)$$

for all  $x, y \in X$ , we have

$$\begin{aligned} \|\psi x - \psi y\|^2 &\leq \|(I - \mu F)x - (I - \mu F)y\|^2 = \|(x - y) + \mu(Fy - Fx)\|^2 \\ &\leq \|x - y\|^2 + 2\langle \mu(Fy - Fx), J((x - y) + \mu(Fy - Fx)) \rangle \\ &\leq \|x - y\|^2 + 2\mu\langle Fy - Fx, J(x - y) \rangle + 2\beta\mu^2\langle Fy - Fx, J(Fy - Fx) \rangle \\ &\leq \|x - y\|^2 - 2\mu\langle Fx - Fy, J(x - y) \rangle + 2\beta\mu^2\|Fy - Fx\| \|J(Fy - Fx)\| \quad (6) \\ &\leq \|x - y\|^2 - 2\mu\eta\|x - y\|^2 + 2\beta\mu^2\|Fy - Fx\|^2 \\ &\leq \|x - y\|^2 - 2\mu\eta\|x - y\|^2 + 2\mu^2\beta\kappa^2\|x - y\|^2 \\ &\leq (1 - 2\mu\eta + 2\mu^2\beta\kappa^2)\|x - y\|^2. \end{aligned}$$

Clearly, the same inequality holds if  $X$  is a 2-uniformly smooth real Banach space. Thus, we obtain

$$\|\psi x - \psi y\| \leq \sqrt{1 - 2\mu(\eta - \mu\beta\kappa^2)} \|x - y\|. \quad (7)$$

With no loss of generality we can take  $\beta \geq 1/2$ ; therefore, if  $\mu \in (0, \eta/\beta\kappa^2)$ , then we have  $\sqrt{1 - 2\mu(\eta - \mu\beta\kappa^2)} \in (0, 1)$ ; that is,  $\psi$  is a contraction, and the proof is complete.  $\square$

Also Lemma 5.1, which is easily proved in the same way as Lemma 3.1, will be as follows.

**Lemma 5.1** (see [1]). *Either let  $X$  be a real Banach space, and let  $J$  be the single-valued normalized duality mapping from  $X$  into  $2^{X^*}$  satisfying (3), or let  $X$  be a 2-uniformly smooth real Banach space. Assume that  $F : X \rightarrow X$  is  $\eta$ -strongly monotone and  $\kappa$ -Lipschitzian on  $X$ . If  $\mu \in (0, \eta/\sigma^2)$ , where  $\sigma = \sqrt{\beta}(\kappa + 2)$ , then*

$$\psi(x) = I(x) - \mu(F + I - T)(x) \quad (8)$$

*is a contraction on  $X$ .*

With the new imposed conditions and considering the above lemmas, the following corrections should be done in [1]:

- (1) in Theorem 3.2 and Theorem 4.2,  $\mu \in (0, \eta/\beta\kappa^2)$ ;
- (2) in Theorem 5.2,  $\mu \in (0, \eta/(\sigma^2 + 1))$ , where  $\sigma = \sqrt{\beta}(\kappa + 2)$ ;
- (3) in Remark 5.3,  $\mu \in (0, 2(\eta - 1)/(2\sigma^2 - 1))$ , where  $\sigma = \sqrt{\beta}(\kappa + 2)$ .

Also in [1, Corollary 4.3] the real Banach space  $X$  does not necessarily need to have a uniformly Gateaux differentiable norm.

To avoid any ambiguity in terminology note also that  $\eta$ -strongly monotone mappings in Banach spaces are usually called  $\eta$ -strongly accretive.

## References

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